

NASA Technical Memorandum 106525

N-24  
329  
p-16

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March 1994

(NASA-TM-106525) A HYDROSTATIC  
STRESS-DEPENDENT ANISOTROPIC MODEL  
OF VISCOPLASTICITY (NASA) 16 p

N94-27640

Unclass



National Aeronautics and  
Space Administration

G3/24 0000329



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## ABSTRACT

A hydrostatic stress-dependent, anisotropic model of viscoplasticity is formulated as an extension of Bodner's model. This represents a further extension of the isotropic Bodner model over that made to anisotropy by Robinson and Miti-Kavuma (1993). Here, account is made of the inelastic deformation that can occur in metallic composites under hydrostatic stress. A procedure for determining the material parameters is identified that is virtually identical to the established characterization procedure for the original Bodner model. Characterization can be achieved using longitudinal / transverse tensile and shear tests and hydrostatic stress tests; alternatively, four off-axis tensile tests can be used. Conditions for a yield stress minimum under off-axis tension are discussed. The model is applied to a W/Cu composite; characterization is made using off-axis tensile data generated at NASA Lewis Research Center (LeRC).

## 1. INTRODUCTION

The model of viscoplasticity introduced by Bodner and Partom (1975) and reviewed by Bodner (1987) is one of the class of unified constitutive models. It applies to isotropic viscoplastic solids and is particularly effective in representing rate-sensitive, non isothermal responses typical of the histograms of rocket engines, the space shuttle main engine (SSME) being a prominent example. A major strength of Bodner's model is its simplicity and the relative ease with which the material parameters can be determined through experiment.

The Bodner model has been extended by Robinson and Miti-Kavuma (1993) to represent anisotropic materials with application to metal matrix composites (MMC's). Their extended model is capable of representing strong initial anisotropy yet is based on a scalar state variable under the assertion that induced anisotropy, through directional or kinematic hardening, is negligible compared to the strong initial anisotropy. Robinson and Miti-Kavuma (1993) account for varying fiber volume fraction through the introduction of a nonlinear rule of mixtures. They apply the anisotropic model to a SiC/Ti composite.

It has been pointed out by Dvorak and Rao (1976), Dvorak (1983) and Rogers (1990), among others, that plastic deformation (and damage) in the matrix of strongly reinforced composites can result from the application of hydrostatic stress. This suggests that the stress dependence in phenomenological constitutive relations for MMC's should be based not solely on deviatoric stress but also on hydrostatic stress, viz,  $I_1 \equiv \text{tr} \tilde{\sigma} \equiv \sigma_{\text{H}}$ . The objective of this paper is to provide a further extension of the isotropic Bodner

theory of viscoplasticity to include not only anisotropy, as in Robinson and Miti-Kavuma (1993), but an appropriate dependence on hydrostatic stress.

Several constitutive models accounting for the influence of hydrostatic stress on inelastic deformation have been proposed, these include Alliot, Boehler and Sawczuk (1977) for anisotropic rock, Doraivelu et al. (1984) for porous powder-metallurgical materials, Rogers (1990) for 'constraint hardening' fiber-reinforced composites and Chan et al. (1992) for deformation and damage in rock salt.

We formulate the anisotropic, hydrostatic stress-dependent model in terms of an effective stress that depends on  $I_1$  and invariants reflecting transverse isotropy, cf., Lance and Robinson (1971) and Robinson and Duffy (1990), thus replacing  $\sqrt{3J_2}$  in the isotropic Bodner model. The multiaxial model is specialized for longitudinal / transverse tension and shear and hydrostatic pressure; these constitute the fundamental or natural stress states (or tests) for the model. Relationships between material constants specific to these natural states and those of the multiaxial model are identified such that the governing equations in each case reduce identically to the form of the uniaxial equations of the isotropic Bodner model. This enables the same well established characterization procedure developed for the original Bodner model to be applied to the present anisotropic model. A complete procedure for determination of the material parameters is specified on this basis. Thus, the extended model retains the simplicity of the Bodner theory in the ease with which the material constants are determined.

An application of the model is made to a unidirectional W/Cu (Tungsten/Copper) composite. Characterization is based on data generated by M.J. Verilli of the NASA /Lewis Research Center; the testing temperature is 260C and the fiber volume fraction is  $\approx 9\%$ .

## 2. THE MULTIAXIAL MODEL

We state the multiaxial form of the anisotropic, hydrostatic stress-dependent model for isothermal conditions. Temperature dependence can be included as in Bodner (1987) and Robinson and Miti-Kavuma (1993). The flow law is

$$\frac{\dot{\epsilon}_{ij}}{\dot{\epsilon}_o} = \frac{3}{2\bar{\sigma}} \exp \left[ -\frac{1}{2} \left( \frac{Z}{\bar{\sigma}} \right)^{2n} \right] \Gamma_{ij} \quad (2.1)$$

and the evolution law is

$$\dot{Z} = m(Z_s - Z) \sigma_{ij} \dot{\epsilon}_{ij} \quad (2.2)$$

in which  $\dot{\epsilon}_{ij}$  are the components of inelastic strain rate,  $\bar{\sigma}$  is the effective stress

$$\bar{\sigma} = \sqrt{3 \left[ J_2 - \xi(J - J_0^2) - (\zeta - \eta)J_0^2 + \frac{1}{9}(\zeta - 4\eta)I_1^2 \right]} \quad (2.3)$$

and  $Z$  is the internal state variable (stress history parameter).

The invariants  $J_2, J_0, J$  and  $I_1$  in (2.3) are defined as

$$J_2 = \frac{1}{2} s_{ij} s_{ji} \quad J_0 = D_{ij} s_{ji} \quad J = D_{ij} s_{jk} s_{ki} \quad I_1 = \sigma_{ii} \quad (2.4)$$

Also,

$$\Gamma_{ij} = s_{ij} - \xi(D_{ki} s_{jk} + D_{jk} s_{ki} - 2J_0 D_{ij}) - 2(\zeta - \eta)J_0(D_{ij} - \frac{1}{3}\delta_{ij}) + \frac{2}{9}(\zeta - 4\eta)I_1\delta_{ij} \quad (2.5)$$

with  $0 \leq \xi < 1 \quad 0 \leq \eta < \frac{\zeta}{4} \quad 0 \leq \zeta < 1$  (2.6)

$\sigma_{ij}$  are the components of stress,  $s_{ij}$  are the components of deviatoric stress and  $D_{ij} \equiv d_i d_j$  is a symmetric orientation tensor, cf., Spencer (1972) and Robinson and Duffy (1990), formed by the product of the components of a unit vector  $d_i$  that designates the local direction of transverse isotropy.

The quantities

$$n, m, \dot{\epsilon}_0, Z_s \text{ and } Z_0 \text{ (the initial value of } Z) \quad (2.7)$$

are the material constants of viscoplasticity;  $\xi, \eta$  and  $\zeta$  are constants specifying the degree of anisotropy and the hydrostatic stress dependence.

### 3. SPECIAL CASES

#### Hydrostatic Stress Independence ( $\zeta - 4\eta \rightarrow 0$ )

The model (2.1)-(2.6) reduces to one insensitive to hydrostatic stress as  $\zeta - 4\eta \rightarrow 0$ . Then, (2.3) and (2.5) become

$$\bar{\sigma} = \sqrt{3 \left[ J_2 - \xi(J - J_0^2) - \frac{3}{4} \zeta J_0^2 \right]} \quad (3.1)$$

and

$$\Gamma_{ij} = s_{ij} - \xi(D_{ki} s_{jk} + D_{jk} s_{ki} - 2J_0 D_{ij}) - \frac{3}{2} \zeta J_0 (D_{ij} - \frac{1}{3} \delta_{ij}) \quad (3.2)$$

which are identical to those proposed earlier in the anisotropic extension of Bodner's model by Robinson and Miti-Kavuma (1993). Hydrostatic stress independence implies inelastic incompressibility, i.e.,

$$\Gamma_{ii} = 0 \quad \text{and} \quad \dot{\varepsilon}_{ii} = 0 \quad (3.3)$$

as is readily shown from (2.1) and (3.2). Further,  $\zeta \rightarrow 1$  in (3.1) and (3.2) corresponds to longitudinal inextensibility, i.e.,

$$D_{ij}\dot{\varepsilon}_{ji} = 0 \quad (3.4)$$

#### Equal Behavior in Longitudinal and Transverse Shear ( $\xi = 0$ )

It is often observed that the inelastic responses in longitudinal and transverse shear are not significantly different in some strongly reinforced metallic composites. The limiting case of these responses being equal corresponds to  $\xi = 0$ , giving

$$\bar{\sigma} = \sqrt{3 \left[ J_2 - (\zeta - \eta)J_0^2 + \frac{1}{9}(\zeta - 4\eta)I_1^2 \right]} \quad (3.5)$$

$$\Gamma_{ij} = s_{ij} - 2(\zeta - \eta)J_0(D_{ij} - \frac{1}{3}\delta_{ij}) + \frac{2}{9}(\zeta - 4\eta)I_1\delta_{ij} \quad (3.6)$$

If, in addition, the material is insensitive to hydrostatic stress ( $\zeta - 4\eta \rightarrow 0$ ), we have

$$\bar{\sigma} = \sqrt{3 \left[ J_2 - \frac{3}{4}\zeta J_0^2 \right]} \quad (3.7)$$

$$\Gamma_{ij} = s_{ij} - \frac{3}{2}\zeta J_0(D_{ij} - \frac{1}{3}\delta_{ij}) \quad (3.8)$$

as used earlier by Robinson and Miti-Kavuma (1993) to characterize a SiC/Ti composite material. Further, taking  $\zeta \rightarrow 1$ , i.e., longitudinal inextensibility, a model analogous to that discussed in Binienda and Robinson (1991) results.

#### Isotropy ( $\xi = \eta = \zeta = 0$ )

In the limiting case of full isotropy, the present model (2.1)-(2.6) reduces to that of Bodner and Partom (1975) and Bodner (1987) with

$$\bar{\sigma} = \sqrt{3J_2} \quad (3.9)$$

$$\Gamma_{ij} = s_{ij} \quad (3.10)$$

#### 4. FUNDAMENTAL STRESS STATES: DEFINITION OF MATERIAL PARAMETERS

The uniaxial form of the isotropic Bodner model is

$$\frac{\dot{\epsilon}}{\dot{\epsilon}_0^*} = \exp \left[ -\frac{1}{2} \left( \frac{Z^*}{\sigma} \right)^{2n} \right] \quad (4.1)$$

$$\dot{Z}^* = m(Z_s^* - Z^*)\sigma\dot{\epsilon} \quad (4.2)$$

cf. Bodner (1987).  $\sigma$  is the uniaxial stress and  $\dot{\epsilon}$  is the inelastic strain rate in the stress direction.

The material constants

$$n, m, \dot{\epsilon}_0^*, Z_s^* \text{ and } Z_0^* \quad (4.3)$$

are found by correlating calculated responses based on (4.1) and (4.2) with experimental data obtained at strain-rates (and temperatures) of interest. The characterization procedure for the Bodner model is well established and discussed in Bodner and Partom (1975) and Bodner (1987).

We now apply the present model (2.1)-(2.6) to the four fundamental stress states shown in Fig. 1a-1d; (a) longitudinal tension (LN), (b) transverse tension (TN), (c) longitudinal shear (LS) and (d) transverse shear (TS). In each case, the flow and evolution laws reduce identically to the form (4.1)-(4.2) provided the following identifications are made relating the state variable  $Z^*$  and the constants  $\dot{\epsilon}_0^*$ ,  $Z_s^*$  and  $Z_0^*$  in (4.1)-(4.2) to  $Z$  and the corresponding constants  $\dot{\epsilon}_0$ ,  $Z_s$  and  $Z_0$  in the multiaxial model (2.1)-(2.6):

(a) Longitudinal tension (LN)

$$Z^* = Z^{LN} = \frac{Z}{\sqrt{1-\zeta}} \quad (4.4)$$

$$\dot{\epsilon}_0^* = \dot{\epsilon}_0^{LN} = \dot{\epsilon}_0 \sqrt{1-\zeta} \quad Z_{s,0}^* = Z_{s,0}^{LN} = \frac{Z_{s,0}}{\sqrt{1-\zeta}} \quad (4.5)$$

(b) Transverse tension (TN)

$$Z^* = Z^{TN} = \frac{Z}{\sqrt{1-\eta}} \quad (4.6)$$

$$\dot{\epsilon}_0^* = \dot{\epsilon}_0^{TN} = \dot{\epsilon}_0 \sqrt{1-\eta} \quad Z_{s,0}^* = Z_{s,0}^{TN} = \frac{Z_{s,0}}{\sqrt{1-\eta}} \quad (4.7)$$

(c) Longitudinal shear (LS)

$$Z^* = Z^{LS} = \frac{Z}{\sqrt{3(1-\xi)}} \quad (4.8)$$

$$\dot{\epsilon}_0^* = \dot{\epsilon}_0^{LS} = \frac{1}{2} \dot{\epsilon}_0 \sqrt{3(1-\xi)} \quad Z_{s,0}^* = Z_{s,0}^{LS} = \frac{Z_{s,0}}{\sqrt{3(1-\xi)}} \quad (4.9)$$

(d) Transverse shear (TS)

$$Z^* = Z^{TS} = \frac{Z}{\sqrt{3}} \quad (4.10)$$

$$\dot{\epsilon}_0^* = \dot{\epsilon}_0^{TS} = \frac{\sqrt{3}}{2} \dot{\epsilon}_0 \quad Z_{s,0}^* = Z_{s,0}^{TS} = \frac{Z_{s,0}}{\sqrt{3}} \quad (4.11)$$

Because the governing equations in each case (a)-(d) reduce identically to the Bodner form (4.1)-(4.2), it follows that the characterization procedure used for the isotropic Bodner model along with uniaxial test data can be applied to the present anisotropic model with test data for the stress states (a)-(d). As indicated earlier, this characterization procedure is well established.

Once the appropriate experiments are conducted under the stress states (a)-(d) and the material constants (4.4)-(4.11) determined from the known Bodner characterization procedure,  $\xi$ ,  $\eta$  and  $\zeta$  are then determined from

$$\xi = 1 - \left( \frac{Z_0^{TS}}{Z_0^{LS}} \right)^2 \quad \eta = 1 - 3 \left( \frac{Z_0^{TS}}{Z_0^{TN}} \right)^2 \quad \zeta = 1 - 3 \left( \frac{Z_0^{TS}}{Z_0^{LN}} \right)^2 \quad (4.12)$$

Another fundamental stress state is that of hydrostatic pressure (Fig. 1e). This, too, leads to governing equations identical in form to (4.1) and (4.2) with  $\sigma$  replaced by the hydrostatic stress  $p$  and  $\dot{\epsilon}$  replaced by  $\dot{\epsilon} = -\dot{\epsilon}_{ii}$ , the rate of volume change. In this case we have

$$Z^* = Z^H = \frac{Z}{\sqrt{3(\zeta - 4\eta)}} \quad (4.13)$$

$$\dot{\epsilon}_0^* = \dot{\epsilon}_0^H = \dot{\epsilon}_0 \sqrt{3(\zeta - 4\eta)} \quad Z_{s,0}^* = Z_{s,0}^H = \frac{Z_{s,0}}{\sqrt{3(\zeta - 4\eta)}} \quad (4.14)$$

leading to

$$\zeta - 4\eta = \left( \frac{Z_0^{TS}}{Z_0^H} \right)^2 \quad (4.15)$$



The hydrostatic test, Fig. 1e, can replace either of the uniaxial tests Fig. 1a or 1b as one of the fundamental tests, with (4.15) then replacing either of the last two equations (4.12).

Although, in principle, the tests of Fig. 1a-1e are the natural ones for the model (2.1)-(2.6), the shear tests (Fig. 1c and 1d) and the hydrostatic test (Fig. 1e) are difficult to conduct and such data are seldom available. In the next section we consider an alternative set of uniaxial tests for determining the material parameters.

## 5. OFF-AXIS UNIAXIAL TESTS : CHARACTERIZATION

We now consider uniaxial tensile stress applied at an arbitrary angle  $\theta$  with the axis of transverse isotropy. Ideally, such data can be generated by axially loading a thin walled tube as shown in Fig. 2.

The governing equations for the response  $\dot{\epsilon}$  along the tensile stress  $\sigma$  direction (axial in Fig. 2) are found from (2.1)-(2.6) and, again, they reduce to the form (4.1)-(4.2), viz.,

$$\frac{\dot{\epsilon}}{\dot{\epsilon}_0} = \exp \left[ -\frac{1}{2} \left( \frac{Z^\theta}{\sigma} \right)^{2n} \right] \quad (5.1)$$

$$\dot{Z}^\theta = m(Z_s^\theta - Z^\theta) \sigma \dot{\epsilon} \quad (5.2)$$

with 
$$Z^\theta = \frac{Z}{\sqrt{f(\theta, \xi, \eta, \zeta)}} \quad (5.3)$$

$$\dot{\epsilon}_0^\theta = \dot{\epsilon}_0 \sqrt{f(\theta, \xi, \eta, \zeta)} \quad Z_{s,0}^\theta = \frac{Z_{s,0}}{\sqrt{f(\theta, \xi, \eta, \zeta)}} \quad (5.4)$$

and 
$$f(\theta, \xi, \eta, \zeta) = 1 - 3\xi \cos^2 \theta \sin^2 \theta + \frac{1}{3}(\eta - \zeta)(2 \cos^2 \theta - \sin^2 \theta)^2 + \frac{1}{3}(\zeta - 4\eta) \quad (5.5)$$

As argued earlier, because the governing equations reduce to the same Bodner form, we can determine the material parameters (5.4) using the known characterization procedure.

We note that  $\theta = 0^\circ$  and  $\theta = 90^\circ$  in (5.1)-(5.5) correspond, respectively, to cases (a) longitudinal tension (LN) and (b) transverse tension (TN) of the previous section.

Now we suppose data has been generated as in Fig. 2 for  $\theta = 0^\circ, \theta_1^\theta, \theta_2^\theta$  and  $90^\circ$ .

We further suppose that  $Z_0^{LN}, Z_0^{TN}, Z_0^{\theta_1}$  and  $Z_0^{\theta_2}$  have been determined using the Bodner characterization procedure. Then, from (4.5), (4.7), (5.4) and (5.5) there results,

$$\left(\frac{Z_0^{LN}}{Z_0^{TN}}\right)^2 = \frac{1-\eta}{1-\zeta} \quad (5.6)$$

$$\left(\frac{Z_0^{LN}}{Z_0^{\theta_1}}\right)^2 = \frac{f(\theta_1, \xi, \eta, \zeta)}{1-\zeta} \quad (5.7)$$

$$\left(\frac{Z_0^{LN}}{Z_0^{\theta_2}}\right)^2 = \frac{f(\theta_2, \xi, \eta, \zeta)}{1-\zeta} \quad (5.8)$$

providing three linear equations for the determination of the anisotropy parameters  $\xi, \eta$  and  $\zeta$ .

The remaining viscoplasticity parameters can be obtained from either of the four testing orientations, e.g.,  $\theta = 0^\circ$  longitudinal tension (LN), in which case we have  $n, m, \dot{\epsilon}_0^{LN}$  and  $Z_s^{LN}$  in addition to  $Z_0^{LN}$ . Using (4.5), the parameters (2.7) are known and the multi-axial model (2.1)-(2.6) is fully specified.

The characterization procedure discussed above is applied to a W/Cu metallic composite in the next section. First, however, we examine some features of the function  $f(\theta, \xi, \eta, \zeta)$  in (5.3)-(5.5).

It is easily shown that  $f(\theta, \xi, \eta, \zeta)$  achieves a maximum at

$$\theta_m = \tan^{-1} \sqrt{\frac{4(\zeta - \eta) - 3\xi}{2(\zeta - \eta) - 3\xi}} \quad (5.9)$$

while, for example,  $Z_0^\theta$  in (5.4) achieves a minimum. As  $Z_0^\theta$  can be considered roughly the yield stress in the stress direction  $\theta$ , it follows that the yield stress is a minimum at  $\theta_m$ .

In the limiting cases

$$4(\zeta - \eta) - 3\xi = 0 \quad (5.10)$$

$$\text{or} \quad 2(\zeta - \eta) - 3\xi = 0 \quad (5.11)$$

$\theta_m = 0^\circ$  or  $90^\circ$ , respectively. For the special case  $\xi = 0$ , cf., (3.5) and (3.6),  $\theta_m \approx 54.73^\circ$  for all  $\zeta$  and  $\eta$ . With

$$4(\zeta - \eta) - 3\xi > 0 \quad (5.12)$$

$$\text{and} \quad 2(\zeta - \eta) - 3\xi < 0 \quad (5.13)$$

$\theta_m$  is imaginary (see Fig. 3) and the yield stress decreases monotonically as  $\theta = 0^\circ \rightarrow 90^\circ$ , with no minimum in  $0^\circ \leq \theta \leq 90^\circ$ .

## 6. APPLICATION TO W/Cu

Application of the model (2.1)-(2.6) is made to a W/Cu (Tungsten /Copper) composite based on data generated by M.J.Verilli of NASA/LeRC. The testing temperature is 260C and the fiber volume fraction is  $\approx 9\%$ . Most of the tensile data used were generated at a strain rate of  $\approx .001/s$ , with additional data at  $\approx .000001/s$  used for establishing the strain rate dependence. Extension to other temperatures and fiber volume fractions follows as in Robinson and Miti-Kavuma (1993).

In addition to longitudinal  $0^\circ$  (LN) and transverse  $90^\circ$  (TN) tensile tests, off-axis tensile tests with  $\theta = 5^\circ$  and  $15^\circ$  were used as part of the data base. The off-axis tests were made not on thin-walled tubes as illustrated in Fig. 2 but instead on laminated cross-ply coupon specimens. We assume that inter laminar shear at these relatively small cross-ply angles has only a second order effect on the off-axis tensile response.

Application of the characterization procedure described in the previous section results in the material parameters characterizing the 9% W/Cu composite at 260C shown in Table 1.

Correlation of tensile data and calculations based on the material parameters of Table 1 is shown in Fig.4. Data and calculations are included for  $\theta = 0^\circ, 5^\circ, 15^\circ$  and  $90^\circ$ ; calculations alone are shown for  $\theta = 30^\circ$  and  $55^\circ$ . As additional data become available for  $\theta = 30^\circ$  and  $55^\circ$ , comparison with the predicted results of Fig. 4 should provide a definitive assessment of the accuracy of the present model regarding off-axis behavior.

The point P in Fig. 3 corresponds to the set of anisotropy parameters  $\xi, \eta$  and  $\zeta$  as specified in Table 1. These were determined by solving (5.6)-(5.8) on a PC using the Levenberg-Marquardt procedure available in Mathcad 4.0. Inserting the values of  $\xi, \eta$  and  $\zeta$  from Table 1 into (5.9) gives  $\theta_m \approx 85^\circ$  as the fiber orientation for minimal yield stress. The predicted tensile curve for  $\theta_m = 85^\circ$  is virtually indistinguishable from that for  $\theta = 90^\circ$  and is not shown in Fig. 4. In fact, the predicted tensile curves for the entire range  $55^\circ < \theta < 90^\circ$  show little change. Corroboration of this feature remains to be made as experimental data become available.

Fig. 5 compares data and calculations for  $0^\circ$  tensile tests at strain rates of .001/s and .000001/s. Evidently, the strain rate dependence is captured adequately over the three decades of strain rate. This rests principally on the choice of the material parameter  $n$  and secondarily on  $\dot{\epsilon}_o$ .

## 7. SUMMARY AND CONCLUSIONS

A hydrostatic stress-dependent, transversely isotropic viscoplasticity model is developed by extending the well known isotropic Bodner model. The resulting model is tractable and retains the simplicity of the Bodner theory, particularly regarding the relative ease with which the material parameters are determined. It is capable of representing strong initial anisotropy yet is based on a single scalar state variable under the assertion that induced anisotropy can be ignored relative to the strong initial anisotropy. Like the original Bodner model, the present one is effective in representing rate-sensitive, non

isothermal viscoplastic responses typical of the histograms of rocket engines, e.g., the SSME. The dependence on  $I_1$  allows for the occurrence of inelastic deformation in the presence of hydrostatic stress as identified by Dvorak and Rao (1976) and others.

Natural stress states (tests) for the model are identified as longitudinal / transverse tension and shear and pressure-volume. Because results of shear and pressure-volume tests are not commonly available, an alternative characterization procedure is proposed using off-axis tensile tests. Conditions on  $\xi, \eta$  and  $\zeta$  are examined for which a yield stress minimum, less than the transverse yield stress, is realized under off-axis tension.

Although the parameters  $\xi, \eta$  and  $\zeta$  are physically based, designating the degree of anisotropy and hydrostatic stress dependence, empirically they play the secondary role of providing flexibility in correlating off-axis tensile response. A recognized limitation of the earlier Robinson and Miti-Kavuma (1993) formulation is that it allows matching of off-axis tensile data only at two orientations, e.g.,  $\theta = 0^\circ$  and  $90^\circ$ . Here, two additional orientations intermediate to  $0^\circ$  and  $90^\circ$  can be fit as well.

Application of the model is made to a unidirectional W/Cu composite having a fiber volume fraction of  $\approx 9\%$ . Characterization is based on tensile tests at 260C with stress / fiber axis orientation angles (Fig. 2) of  $\theta = 0^\circ, 5^\circ, 15^\circ$  and  $90^\circ$ . Good correlation of data and calculations is achieved, both in terms of off-axis tensile response and strain-rate dependence. Predictions of tensile response at other off-axis angles, viz.,  $\theta = 30^\circ$  and  $55^\circ$ , are given. Comparison of these predictions and data as they become available will provide an assessment of the accuracy of the model.

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## 9. ACKNOWLEDGMENT

This research was supported under NASA grant NAG-3 379.

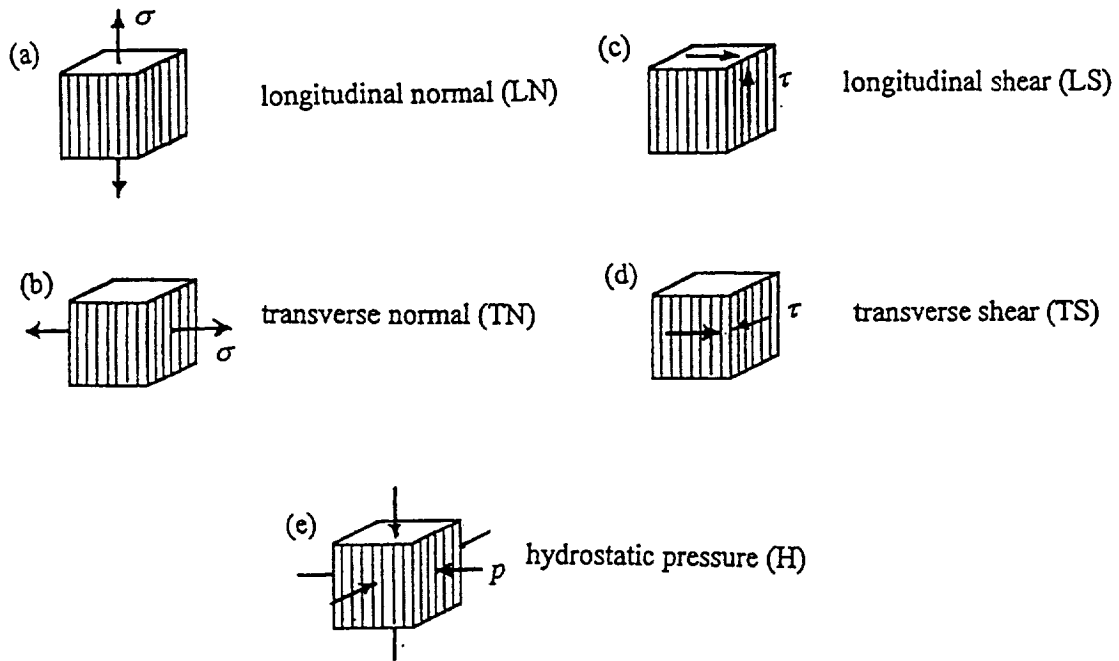


Fig. 1 Fundamental stress states. (a) Longitudinal normal (tension) (LN), (b) Transverse normal (tension) (TN), (c) Longitudinal shear (LS), (d) Transverse shear (TS) and (e) Hydrostatic pressure (H).

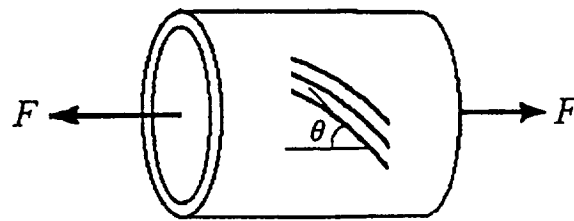


Fig. 2 Axial loading of a thin-walled tubular specimen with helical fibers.  $\theta$  denotes the angle between the stress and fiber directions.

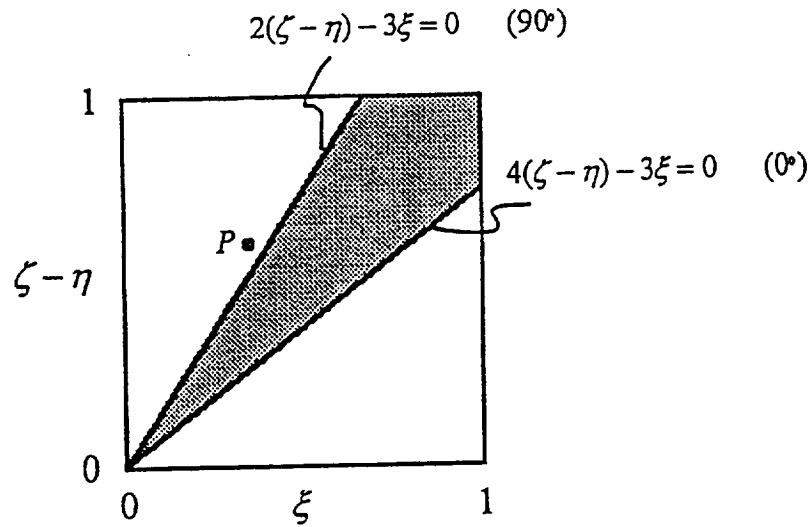


Fig. 3  $\zeta - \eta$  vs.  $\xi$  showing the limiting cases for which  $\theta_m = 0^\circ$  and  $90^\circ$ . The shaded area indicates the locus of points  $\zeta, \eta, \xi$  for which  $\theta_m$  is imaginary, meaning that there is no yield stress minimum between  $0^\circ$  and  $90^\circ$ .

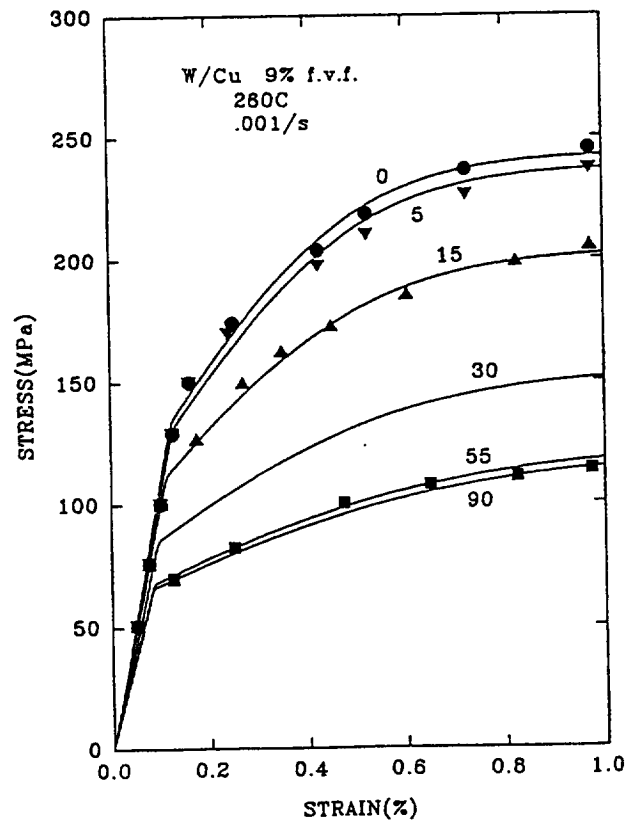


Fig. 4 Correlation of tensile data (symbols) and calculations (lines) based on parameters of Table 1. Data and calculations are shown for  $\theta = 0^\circ, 5^\circ, 15^\circ$  and  $90^\circ$ , calculations alone for  $\theta = 30^\circ$  and  $55^\circ$ . Strain rate is  $\approx .001/s$  and the temperature is  $260^\circ\text{C}$ .

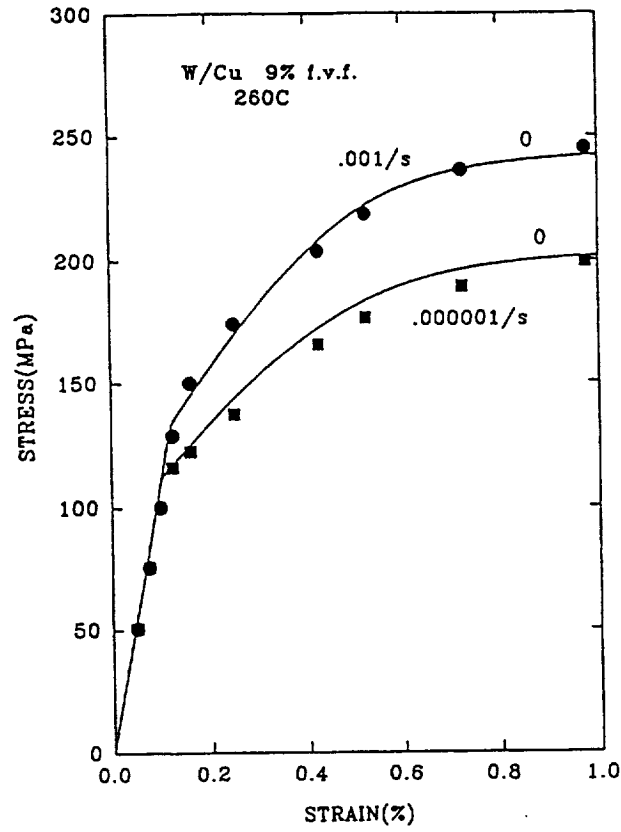


Fig. 5 Comparison of data (symbols) and calculations (lines) for  $0^\circ$  tensile tests at strain rates of 0.001/s and 0.000001/s.

Table 1.- Material Constants for 9% W/Cu at 260C

$n$	1.03
$m$	2.80
$\dot{\epsilon}_0$	$1.28 \times 10^4 (s^{-1})$
$Z_0$	$325 (MPa)$
$Z_s$	$587 (MPa)$
$\xi$	0.364
$\eta$	0.183
$\zeta$	0.793





REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
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1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE March 1994	3. REPORT TYPE AND DATES COVERED Technical Memorandum		
4. TITLE AND SUBTITLE  A Hydrostatic Stress-Dependent Anisotropic Model of Viscoplasticity		5. FUNDING NUMBERS  WU-505-63-12		
6. AUTHOR(S)  D.N. Robinson, Q. Tao, and M. J. Verrilli				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		8. PERFORMING ORGANIZATION REPORT NUMBER  E-8651		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Washington, D.C. 20546-0001		10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA TM-106525		
11. SUPPLEMENTARY NOTES D.N. Robinson and Q. Tao, University of Akron, Akron, Ohio 44325-3905 (work funded by NASA Grant NAG3-379) and M.J. Verrilli, NASA Lewis Research Center. Responsible person, M.J. Verrilli, organization code 5220, (216) 433-3337.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Categories 24 and 39		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) A hydrostatic stress-dependent, anisotropic model of viscoplasticity is formulated as an extension of Bodner's model. This represents a further extension of the isotropic Bodner model over that made to anisotropy by Robinson and Miti-Kavuma (1993). Here, account is made of the inelastic deformation that can occur in metallic composites under hydrostatic stress. A procedure for determining the material parameters is identified that is virtually identical to the established characterization procedure for the original Bodner model. Characterization can be achieved using longitudinal/transverse tensile and shear tests and hydrostatic stress tests; alternatively, four off-axis tensile tests can be used. Conditions for a yield stress minimum under off-axis tension are discussed. The model is applied to a W/Cu composite; characterization is made using off-axis tensile data generated at NASA Lewis Research Center (LeRC).				
14. SUBJECT TERMS  Viscoplasticity; Metal matrix composites; Anisotropic; Hydrostatic stress		15. NUMBER OF PAGES 16		
		16. PRICE CODE A03		
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	